

## DIFFERENTIATION RULES

### GENERAL FORMULAS

$$1. \frac{d}{dx}(c) = 0$$

$$3. \frac{d}{dx}[f(x) + g(x)] = f'(x) + g'(x)$$

$$5. \frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x) \quad (\text{Product Rule})$$

$$7. \frac{d}{dx}f(g(x)) = f'(g(x))g'(x) \quad (\text{Chain Rule})$$

$$2. \frac{d}{dx}[cf(x)] = cf'(x)$$

$$4. \frac{d}{dx}[f(x) - g(x)] = f'(x) - g'(x)$$

$$6. \frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2} \quad (\text{Quotient Rule})$$

$$8. \frac{d}{dx}(x^n) = nx^{n-1} \quad (\text{Power Rule})$$

### EXPONENTIAL AND LOGARITHMIC FUNCTIONS

$$9. \frac{d}{dx}(e^x) = e^x$$

$$11. \frac{d}{dx} \ln|x| = \frac{1}{x}$$

$$10. \frac{d}{dx}(a^x) = a^x \ln a$$

$$12. \frac{d}{dx}(\log_a x) = \frac{1}{x \ln a}$$

### TRIGONOMETRIC FUNCTIONS

$$13. \frac{d}{dx}(\sin x) = \cos x$$

$$16. \frac{d}{dx}(\csc x) = -\csc x \cot x$$

$$14. \frac{d}{dx}(\cos x) = -\sin x$$

$$17. \frac{d}{dx}(\sec x) = \sec x \tan x$$

$$15. \frac{d}{dx}(\tan x) = \sec^2 x$$

$$18. \frac{d}{dx}(\cot x) = -\csc^2 x$$

### INVERSE TRIGONOMETRIC FUNCTIONS

$$19. \frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$$

$$22. \frac{d}{dx}(\csc^{-1} x) = -\frac{1}{x\sqrt{x^2-1}}$$

$$20. \frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$$

$$23. \frac{d}{dx}(\sec^{-1} x) = \frac{1}{x\sqrt{x^2-1}}$$

$$21. \frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$$

$$24. \frac{d}{dx}(\cot^{-1} x) = -\frac{1}{1+x^2}$$

### HYPERBOLIC FUNCTIONS

$$25. \frac{d}{dx}(\sinh x) = \cosh x$$

$$28. \frac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$$

$$26. \frac{d}{dx}(\cosh x) = \sinh x$$

$$29. \frac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$$

$$27. \frac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$$

$$30. \frac{d}{dx}(\operatorname{coth} x) = -\operatorname{csch}^2 x$$

### INVERSE HYPERBOLIC FUNCTIONS

$$31. \frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{1+x^2}}$$

$$34. \frac{d}{dx}(\operatorname{csch}^{-1} x) = -\frac{1}{|x|\sqrt{x^2+1}}$$

$$32. \frac{d}{dx}(\cosh^{-1} x) = \frac{1}{\sqrt{x^2-1}}$$

$$35. \frac{d}{dx}(\operatorname{sech}^{-1} x) = -\frac{1}{x\sqrt{1-x^2}}$$

$$33. \frac{d}{dx}(\tanh^{-1} x) = \frac{1}{1-x^2}$$

$$36. \frac{d}{dx}(\operatorname{coth}^{-1} x) = \frac{1}{1-x^2}$$

# TABLE OF INTEGRALS

## BASIC FORMS

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|---|--|--|---|
| 1. $\int u dv = uv - \int v du$                             | 6. $\int \sin u du = -\cos u + C$        | 11. $\int \csc u \cot u du = -\csc u + C$        | 16. $\int \frac{du}{\sqrt{a^2 - u^2}} = \sin^{-1} \frac{u}{a} + C$                    |
| 2. $\int u^n du = \frac{1}{n+1} u^{n+1} + C$<br>$n \neq -1$ | 7. $\int \cos u du = \sin u + C$         | 12. $\int \tan u du = \ln  \sec u  + C$          | 17. $\int \frac{du}{a^2 + u^2} = \frac{1}{a} \tan^{-1} \frac{u}{a} + C$               |
| 3. $\int \frac{du}{u} = \ln  u  + C$                        | 8. $\int \sec^2 u du = \tan u + C$       | 13. $\int \cot u du = \ln  \sin u  + C$          | 18. $\int \frac{du}{u \sqrt{u^2 - a^2}} = \frac{1}{a} \sec^{-1} \frac{u}{a} + C$      |
| 4. $\int e^u du = e^u + C$                                  | 9. $\int \csc^2 u du = -\cot u + C$      | 14. $\int \sec u du = \ln  \sec u + \tan u  + C$ | 19. $\int \frac{du}{a^2 - u^2} = \frac{1}{2a} \ln \left  \frac{a+u}{a-u} \right  + C$ |
| 5. $\int a^u du = \frac{1}{\ln a} a^u + C$                  | 10. $\int \sec u \tan u du = \sec u + C$ | 15. $\int \csc u du = \ln  \csc u - \cot u  + C$ | 20. $\int \frac{du}{u^2 - a^2} = \frac{1}{2a} \ln \left  \frac{u-a}{u+a} \right  + C$ |

## FORMS INVOLVING $\sqrt{a^2 + u^2}$ , $a > 0$

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|--|---|
| 21. $\int \sqrt{a^2 + u^2} du = \frac{u}{2} \sqrt{a^2 + u^2} + \frac{a^2}{2} \ln(u + \sqrt{a^2 + u^2}) + C$                  | 26. $\int \frac{u^2 du}{\sqrt{a^2 + u^2}} = \frac{u}{2} \sqrt{a^2 + u^2} - \frac{a^2}{2} \ln(u + \sqrt{a^2 + u^2}) + C$ |
| 22. $\int u^2 \sqrt{a^2 + u^2} du = \frac{u}{8} (a^2 + 2u^2) \sqrt{a^2 + u^2} - \frac{a^4}{8} \ln(u + \sqrt{a^2 + u^2}) + C$ | 27. $\int \frac{du}{u \sqrt{a^2 + u^2}} = -\frac{1}{a} \ln \left  \frac{\sqrt{a^2 + u^2} + a}{u} \right  + C$           |
| 23. $\int \frac{\sqrt{a^2 + u^2}}{u} du = \sqrt{a^2 + u^2} - a \ln \left  \frac{u + \sqrt{a^2 + u^2}}{u} \right  + C$        | 28. $\int \frac{du}{u^2 \sqrt{a^2 + u^2}} = -\frac{\sqrt{a^2 + u^2}}{a^2 u} + C$  |
| 24. $\int \frac{\sqrt{a^2 + u^2}}{u^2} du = -\frac{\sqrt{a^2 + u^2}}{u} + \ln(u + \sqrt{a^2 + u^2}) + C$                     | 29. $\int \frac{du}{(a^2 - u^2)^{3/2}} = \frac{u}{a^2 \sqrt{a^2 - u^2}} + C$  |
| 25. $\int \frac{du}{\sqrt{a^2 + u^2}} = \ln(u + \sqrt{a^2 + u^2}) + C$   |   |

## FORMS INVOLVING $\sqrt{a^2 - u^2}$ , $a > 0$

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|--|--|
| 30. $\int \sqrt{a^2 - u^2} du = \frac{u}{2} \sqrt{a^2 - u^2} + \frac{a^2}{2} \sin^{-1} \frac{u}{a} + C$                  | 35. $\int \frac{du}{u \sqrt{a^2 - u^2}} = -\frac{1}{a} \ln \left  \frac{a + \sqrt{a^2 - u^2}}{u} \right  + C$            |
| 31. $\int u^2 \sqrt{a^2 - u^2} du = \frac{u}{8} (2u^2 - a^2) \sqrt{a^2 - u^2} + \frac{a^4}{8} \sin^{-1} \frac{u}{a} + C$ | 36. $\int \frac{du}{u^2 \sqrt{a^2 - u^2}} = -\frac{1}{a^2 u} \sqrt{a^2 - u^2} + C$                                       |
| 32. $\int \frac{\sqrt{a^2 - u^2}}{u} du = \sqrt{a^2 - u^2} - a \ln \left  \frac{a + \sqrt{a^2 - u^2}}{u} \right  + C$    | 37. $\int (a^2 - u^2)^{3/2} du = -\frac{u}{8} (2u^2 - 5a^2) \sqrt{a^2 - u^2} + \frac{3a^4}{8} \sin^{-1} \frac{u}{a} + C$ |
| 33. $\int \frac{\sqrt{a^2 - u^2}}{u^2} du = -\frac{1}{u} \sqrt{a^2 - u^2} - \sin^{-1} \frac{u}{a} + C$                   | 38. $\int \frac{du}{(a^2 - u^2)^{3/2}} = \frac{u}{a^2 \sqrt{a^2 - u^2}} + C$   |
| 34. $\int \frac{u^2 du}{\sqrt{a^2 - u^2}} = -\frac{u}{2} \sqrt{a^2 - u^2} + \frac{a^2}{2} \sin^{-1} \frac{u}{a} + C$     |  |

## FORMS INVOLVING $\sqrt{u^2 - a^2}$ , $a > 0$

- |   |  |
|---|--|
| 39. $\int \sqrt{u^2 - a^2} du = \frac{u}{2} \sqrt{u^2 - a^2} - \frac{a^2}{2} \ln  u + \sqrt{u^2 - a^2}  + C$                  | 43. $\int \frac{du}{\sqrt{u^2 - a^2}} = \ln  u + \sqrt{u^2 - a^2}  + C$  |
| 40. $\int u^2 \sqrt{u^2 - a^2} du = \frac{u}{8} (2u^2 - a^2) \sqrt{u^2 - a^2} - \frac{a^4}{8} \ln  u + \sqrt{u^2 - a^2}  + C$ | 44. $\int \frac{u^2 du}{\sqrt{u^2 - a^2}} = \frac{u}{2} \sqrt{u^2 - a^2} + \frac{a^2}{2} \ln  u + \sqrt{u^2 - a^2}  + C$ |
| 41. $\int \frac{\sqrt{u^2 - a^2}}{u} du = \sqrt{u^2 - a^2} - a \cos^{-1} \frac{a}{ u } + C$                                   | 45. $\int \frac{du}{u^2 \sqrt{u^2 - a^2}} = \frac{\sqrt{u^2 - a^2}}{a^2 u} + C$  |
| 42. $\int \frac{\sqrt{u^2 - a^2}}{u^2} du = -\frac{\sqrt{u^2 - a^2}}{u} + \ln  u + \sqrt{u^2 - a^2}  + C$                     | 46. $\int \frac{du}{(u^2 - a^2)^{3/2}} = -\frac{u}{a^2 \sqrt{u^2 - a^2}} + C$  |

# TABLE OF INTEGRALS

## FORMS INVOLVING $a + bu$

7.  $\int \frac{u \, du}{a + bu} = \frac{1}{b^2}(a + bu - a \ln|a + bu|) + C$
8.  $\int \frac{u^2 \, du}{a + bu} = \frac{1}{2b^2}[(a + bu)^2 - 4a(a + bu) + 2a^2 \ln|a + bu|] + C$
9.  $\int \frac{du}{u(a + bu)} = \frac{1}{a} \ln \left| \frac{a}{a + bu} \right| + C$
10.  $\int \frac{du}{u^2(a + bu)} = -\frac{1}{au} + \frac{b}{a^2} \ln \left| \frac{a + bu}{u} \right| + C$
11.  $\int \frac{u \, du}{(a + bu)^2} = \frac{a}{b^2(a + bu)} + \frac{1}{b^2} \ln|a + bu| + C$
12.  $\int \frac{du}{u(a + bu)^2} = \frac{1}{a(a + bu)} - \frac{1}{a^2} \ln \left| \frac{a + bu}{u} \right| + C$
13.  $\int \frac{u^2 \, du}{(a + bu)^2} = \frac{1}{b^3} \left( a + bu - \frac{a^2}{a + bu} - 2a \ln|a + bu| \right) + C$
14.  $\int u \sqrt{a + bu} \, du = \frac{2}{15b^2}(3bu - 2a)(a + bu)^{3/2} + C$
15.  $\int \frac{u \, du}{\sqrt{a + bu}} = \frac{2}{3b^2}(bu - 2a)\sqrt{a + bu} + C$
16.  $\int \frac{u^2 \, du}{\sqrt{a + bu}} = \frac{2}{15b^2}(8a^2 + 3b^2u^2 - 4abu)\sqrt{a + bu} + C$
17.  $\int \frac{du}{u\sqrt{a + bu}} = \frac{1}{\sqrt{a}} \ln \left| \frac{\sqrt{a + bu} - \sqrt{a}}{\sqrt{a + bu} + \sqrt{a}} \right| + C, \text{ if } a > 0$   
 $= \frac{2}{\sqrt{-a}} \tan^{-1} \sqrt{\frac{a + bu}{-a}} + C, \text{ if } a < 0$
18.  $\int \frac{\sqrt{a + bu}}{u} \, du = 2\sqrt{a + bu} + a \int \frac{du}{u\sqrt{a + bu}}$
19.  $\int \frac{\sqrt{a + bu}}{u^2} \, du = -\frac{\sqrt{a + bu}}{u} + \frac{b}{2} \int \frac{du}{u\sqrt{a + bu}}$
20.  $\int u^n \sqrt{a + bu} \, du = \frac{2}{b(2n + 3)} \left[ u^n(a + bu)^{3/2} - na \int u^{n-1} \sqrt{a + bu} \, du \right]$
21.  $\int \frac{u^n \, du}{\sqrt{a + bu}} = \frac{2u^n \sqrt{a + bu}}{b(2n + 1)} - \frac{2na}{b(2n + 1)} \int \frac{u^{n-1} \, du}{\sqrt{a + bu}}$
22.  $\int \frac{du}{u^n \sqrt{a + bu}} = -\frac{\sqrt{a + bu}}{a(n-1)u^{n-1}} - \frac{b(2n-3)}{2a(n-1)} \int \frac{du}{u^{n-1} \sqrt{a + bu}}$

## TRIGONOMETRIC FORMS

1.  $\int \sin^2 u \, du = \frac{1}{2}u - \frac{1}{4}\sin 2u + C$
2.  $\int \cos^2 u \, du = \frac{1}{2}u + \frac{1}{4}\sin 2u + C$
3.  $\int \tan^2 u \, du = \tan u - u + C$
4.  $\int \cot^2 u \, du = -\cot u - u + C$
5.  $\int \sin^3 u \, du = -\frac{1}{3}(2 + \sin^2 u) \cos u + C$
6.  $\int \cos^3 u \, du = \frac{1}{3}(2 - \cos^2 u) \sin u + C$
7.  $\int \tan^3 u \, du = \frac{1}{2} \tan^2 u + \ln|\cos u| + C$
8.  $\int \cot^3 u \, du = -\frac{1}{2} \cot^2 u - \ln|\sin u| + C$
9.  $\int \sec^3 u \, du = \frac{1}{2} \sec u \tan u + \frac{1}{2} \ln|\sec u + \tan u| + C$
10.  $\int \csc^3 u \, du = -\frac{1}{2} \csc u \cot u + \frac{1}{2} \ln|\csc u - \cot u| + C$
11.  $\int \sin^n u \, du = -\frac{1}{n} \sin^{n-1} u \cos u + \frac{n-1}{n} \int \sin^{n-2} u \, du$
12.  $\int \cos^n u \, du = \frac{1}{n} \cos^{n-1} u \sin u + \frac{n-1}{n} \int \cos^{n-2} u \, du$
13.  $\int \tan^n u \, du = \frac{1}{n-1} \tan^{n-1} u - \int \tan^{n-2} u \, du$
14.  $\int \cot^n u \, du = \frac{-1}{n-1} \cot^{n-1} u - \int \cot^{n-2} u \, du$
15.  $\int \sec^n u \, du = \frac{1}{n-1} \tan u \sec^{n-2} u + \frac{n-2}{n-1} \int \sec^{n-2} u \, du$
16.  $\int \csc^n u \, du = \frac{-1}{n-1} \cot u \csc^{n-2} u + \frac{n-2}{n-1} \int \csc^{n-2} u \, du$
17.  $\int \sin au \sin bu \, du = \frac{\sin(a-b)u}{2(a-b)} - \frac{\sin(a+b)u}{2(a+b)} + C$
18.  $\int \cos au \cos bu \, du = \frac{\sin(a-b)u}{2(a-b)} + \frac{\sin(a+b)u}{2(a+b)} + C$
19.  $\int \sin au \cos bu \, du = -\frac{\cos(a-b)u}{2(a-b)} - \frac{\cos(a+b)u}{2(a+b)} + C$
20.  $\int a \sin u \, du = \sin u - u \cos u + C$
21.  $\int u \cos u \, du = \cos u + u \sin u + C$
22.  $\int u^n \sin u \, du = -u^n \cos u + n \int u^{n-1} \cos u \, du$
23.  $\int u^n \cos u \, du = u^n \sin u - n \int u^{n-1} \sin u \, du$
24.  $\int \sin^m u \cos^n u \, du = -\frac{\sin^{m-1} u \cos^{m+1} u}{m+1} + \frac{n-1}{m+1} \int \sin^{m-2} u \cos^n u \, du$   
 $= \frac{\sin^{m-1} u \cos^{m-1} u}{m+1} + \frac{m-1}{m+1} \int \sin^m u \cos^{m-2} u \, du$

## INVERSE TRIGONOMETRIC FORMS

$$87. \int \sin^{-1} u \, du = u \sin^{-1} u + \sqrt{1-u^2} + C$$

$$88. \int \cos^{-1} u \, du = u \cos^{-1} u - \sqrt{1-u^2} + C$$

$$89. \int \tan^{-1} u \, du = u \tan^{-1} u - \frac{1}{2} \ln(1+u^2) + C$$

$$90. \int u \sin^{-1} u \, du = \frac{2u^2-1}{4} \sin^{-1} u + \frac{u\sqrt{1-u^2}}{4} + C$$

$$91. \int u \cos^{-1} u \, du = \frac{2u^2-1}{4} \cos^{-1} u - \frac{u\sqrt{1-u^2}}{4} + C$$

$$92. \int u \tan^{-1} u \, du = \frac{u^2+1}{2} \tan^{-1} u - \frac{u}{2} + C$$

$$93. \int u^n \sin^{-1} u \, du = \frac{1}{n+1} \left[ u^{n+1} \sin^{-1} u - \int \frac{u^{n+1} du}{\sqrt{1-u^2}} \right], n \neq -1$$

$$94. \int u^n \cos^{-1} u \, du = \frac{1}{n+1} \left[ u^{n+1} \cos^{-1} u + \int \frac{u^{n+1} du}{\sqrt{1-u^2}} \right], n \neq -1$$

$$95. \int u^n \tan^{-1} u \, du = \frac{1}{n+1} \left[ u^{n+1} \tan^{-1} u - \int \frac{u^{n+1} du}{1+u^2} \right], n \neq -1$$

## EXPONENTIAL AND LOGARITHMIC FORMS

$$96. \int u e^{au} \, du = \frac{1}{a^2} (au-1)e^{au} + C$$

$$97. \int u^n e^{au} \, du = \frac{1}{a} u^n e^{au} - \frac{n}{a} \int u^{n-1} e^{au} \, du$$

$$98. \int e^{au} \sin bu \, du = \frac{e^{au}}{a^2+b^2} (a \sin bu - b \cos bu) + C$$

$$99. \int e^{au} \cos bu \, du = \frac{e^{au}}{a^2+b^2} (a \cos bu + b \sin bu) + C$$

$$100. \int \ln u \, du = u \ln u - u + C$$

$$101. \int u^n \ln u \, du = \frac{u^{n+1}}{(n+1)^2} [(n+1) \ln u - 1] + C$$

$$102. \int \frac{1}{u \ln u} \, du = \ln |\ln u| + C$$

## HYPERBOLIC FORMS

$$103. \int \sinh u \, du = \cosh u + C$$

$$104. \int \cosh u \, du = \sinh u + C$$

$$105. \int \tanh u \, du = \ln \cosh u + C$$

$$106. \int \coth u \, du = \ln |\sinh u| + C$$

$$107. \int \operatorname{sech} u \, du = \tan^{-1} |\sinh u| + C$$

$$108. \int \operatorname{csch} u \, du = \ln \left| \tanh \frac{1}{2} u \right| + C$$

$$109. \int \operatorname{sech}^2 u \, du = \tanh u + C$$

$$110. \int \operatorname{csch}^2 u \, du = -\coth u + C$$

$$111. \int \operatorname{sech} u \tanh u \, du = -\operatorname{sech} u + C$$

$$112. \int \operatorname{csch} u \coth u \, du = -\operatorname{csch} u + C$$

## FORMS INVOLVING $\sqrt{2au-u^2}$ , $a > 0$

$$113. \int \sqrt{2au-u^2} \, du = \frac{u-a}{2} \sqrt{2au-u^2} + \frac{a^2}{2} \cos^{-1} \left( \frac{a-u}{a} \right) + C$$

$$114. \int u \sqrt{2au-u^2} \, du = \frac{2u^3-au-3a^2}{6} \sqrt{2au-u^2} + \frac{a^3}{2} \cos^{-1} \left( \frac{a-u}{a} \right) + C$$

$$115. \int \frac{\sqrt{2au-u^2}}{u} \, du = \sqrt{2au-u^2} + a \cos^{-1} \left( \frac{a-u}{a} \right) + C$$

$$116. \int \frac{\sqrt{2au-u^2}}{u^2} \, du = -\frac{2\sqrt{2au-u^2}}{u} - \cos^{-1} \left( \frac{a-u}{a} \right) + C$$

$$117. \int \frac{du}{\sqrt{2au-u^2}} = \cos^{-1} \left( \frac{a-u}{a} \right) + C$$

$$118. \int \frac{u \, du}{\sqrt{2au-u^2}} = -\sqrt{2au-u^2} + a \cos^{-1} \left( \frac{a-u}{a} \right) + C$$

$$119. \int \frac{u^2 \, du}{\sqrt{2au-u^2}} = -\frac{(u+3a)}{2} \sqrt{2au-u^2} + \frac{3a^2}{2} \cos^{-1} \left( \frac{a-u}{a} \right) + C$$

$$120. \int \frac{du}{u\sqrt{2au-u^2}} = -\frac{\sqrt{2au-u^2}}{au} + C$$